Electric Potential for Point Charge

Recall that a point charge Q, **located at the origin** ($\bar{r}'=0$), produces a static electric field:

$$\mathbf{E}(\bar{r}) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$$

Now, we know that this field is the **gradient** of some scalar field:

$$\mathbf{E}(\bar{r}) = -\nabla V(\bar{r})$$

Q: What is the **electric potential** function $V(\bar{r})$ generated by a **point charge** Q, located at the origin?

A: We find that it is:

$$V(\bar{r}) = \frac{Q}{4\pi\varepsilon_0 r}$$

Q: *Where did **this** come from ? How do we know that this is the correct solution?*

A: We can show it is the correct solution by **direct substitution**!

$$
\begin{aligned}
\mathbf{E}(\bar{\mathrm{r}}) &= -\nabla V(\bar{\mathrm{r}}) \\
&= -\nabla\left(\frac{Q}{4\pi\varepsilon_0\, r}\right) \\
&= -\frac{\partial}{\partial r}\left(\frac{Q}{4\pi\varepsilon_0\, r}\right)\hat{a}_r + 0 \\
&= \frac{Q}{4\pi\varepsilon_0\, r^2}\,\hat{a}_r
\end{aligned}
$$

The **correct** result!

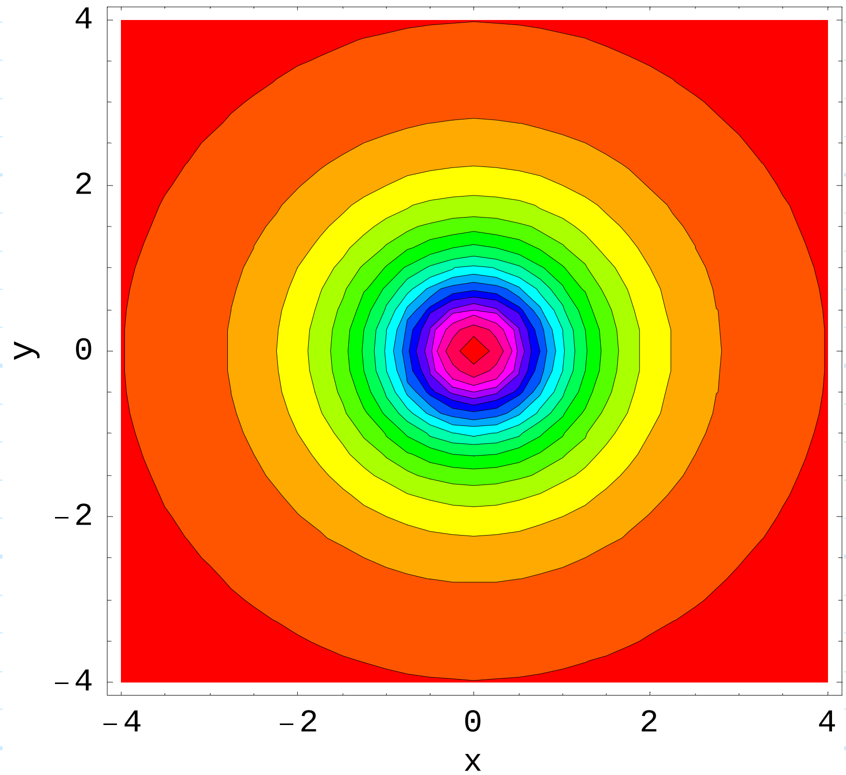
Q: *What if the charge is* ***not*** *located at the* ***origin****?*

A: **Substitute** r with $|\bar{\mathrm{r}}-\bar{\mathrm{r}}'|$, and we get:

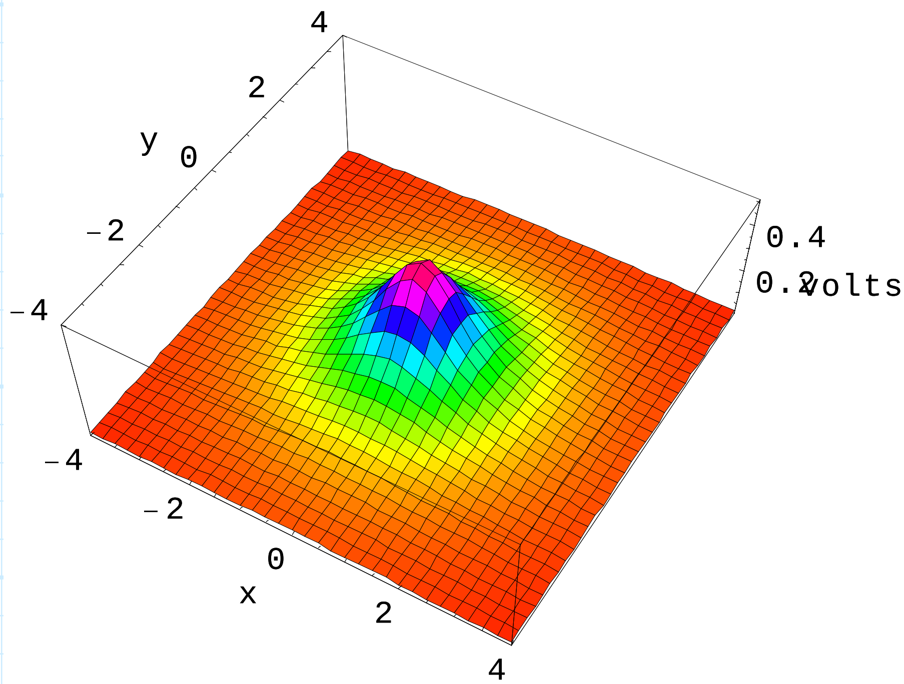
$$
V(\bar{\mathrm{r}}) = \frac{Q}{4\pi\varepsilon_0\, |\bar{\mathrm{r}}-\bar{\mathrm{r}}'|}
$$

Where, as before, the position vector $\bar{\mathrm{r}}'$ denotes the location of the **charge** Q, and the position vector $\bar{\mathrm{r}}$ denotes the location in space where the electric potential function is **evaluated**.

The **scalar** function $V(\bar{r})$ for a point charge can be shown graphically as a **contour plot**:

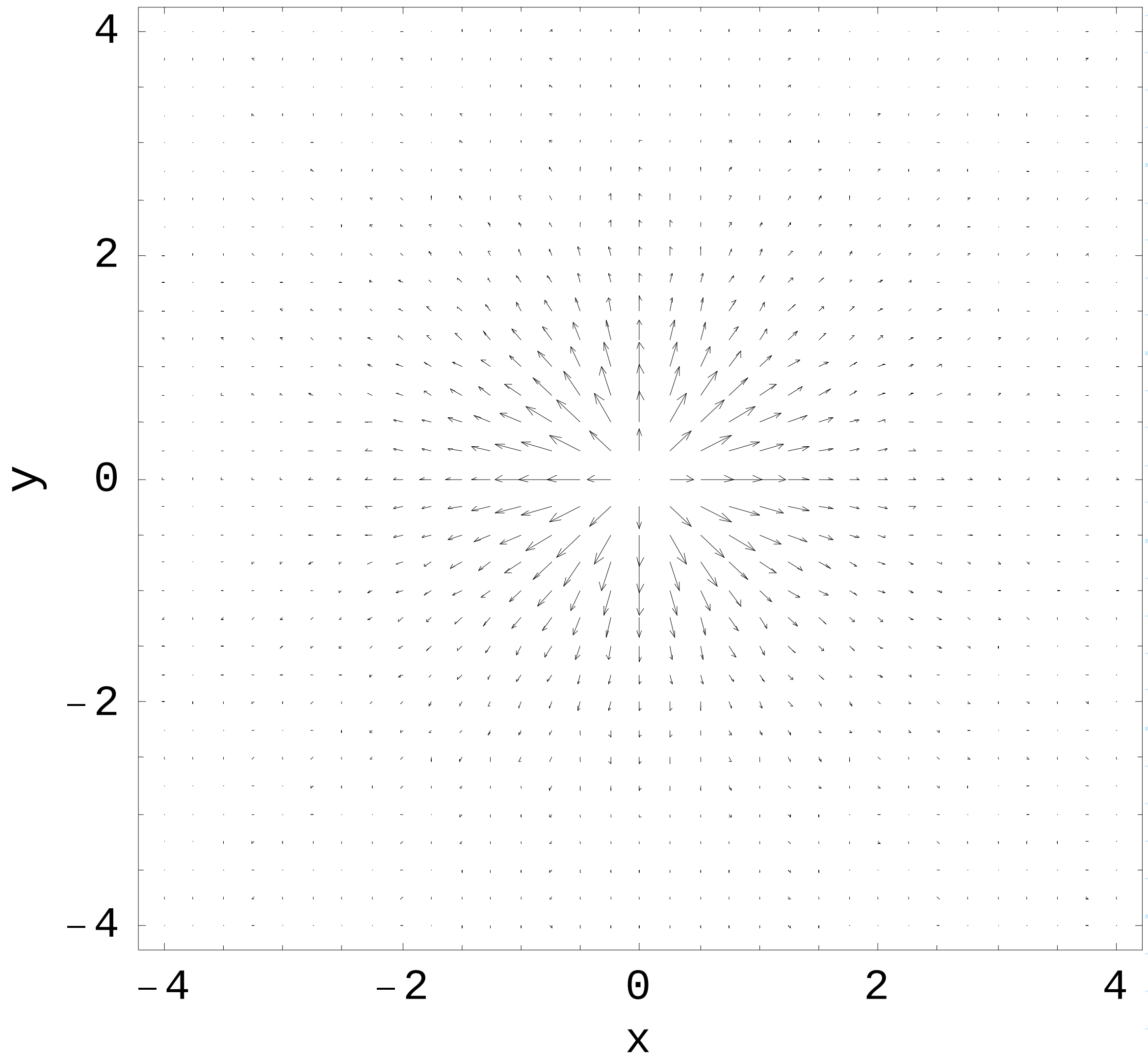


Or, in **three** dimensions as:

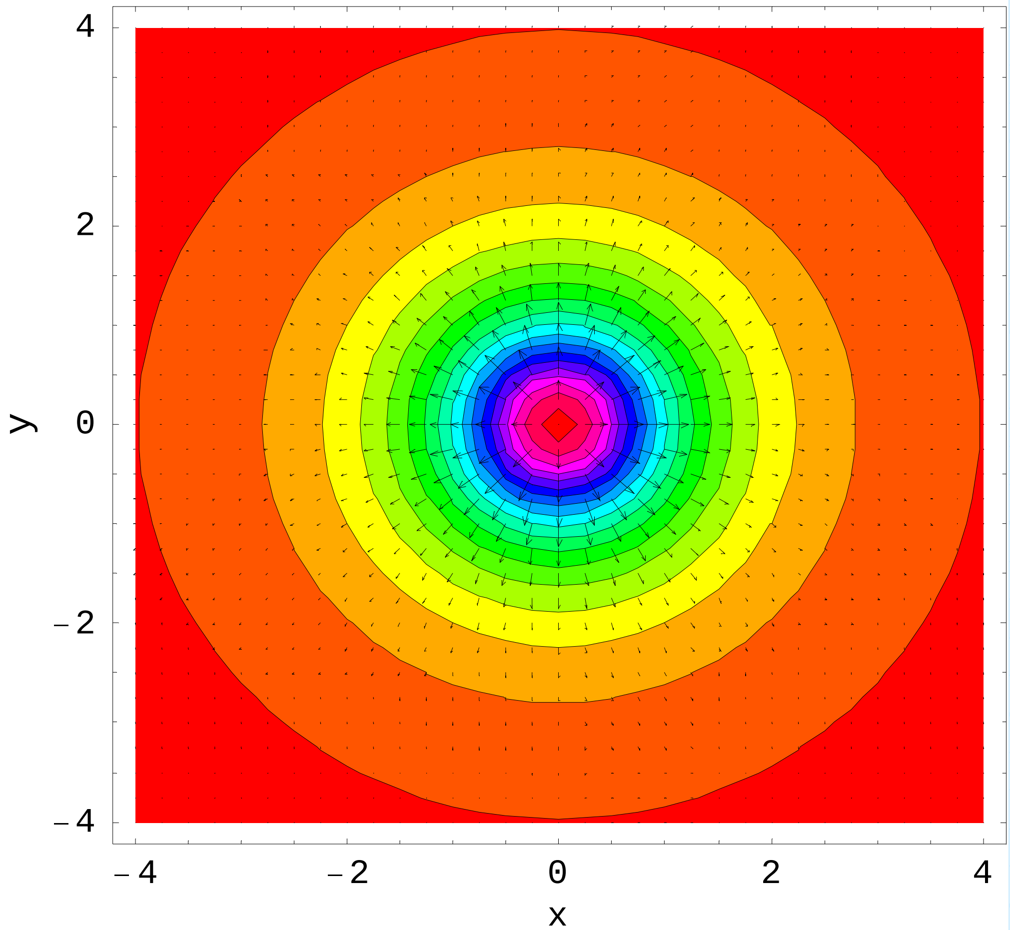


Note the electric potential **increases** as we get **closer** to the point charge (located at the origin). It appears that we have "**mountain**" of electric potential; an appropriate analogy, considering that the potential energy of a mass in the Earth's gravitational field increases with altitude (i.e., height)!

Recall the **electric field** produced by a point charge is a **vector** field that looks like:

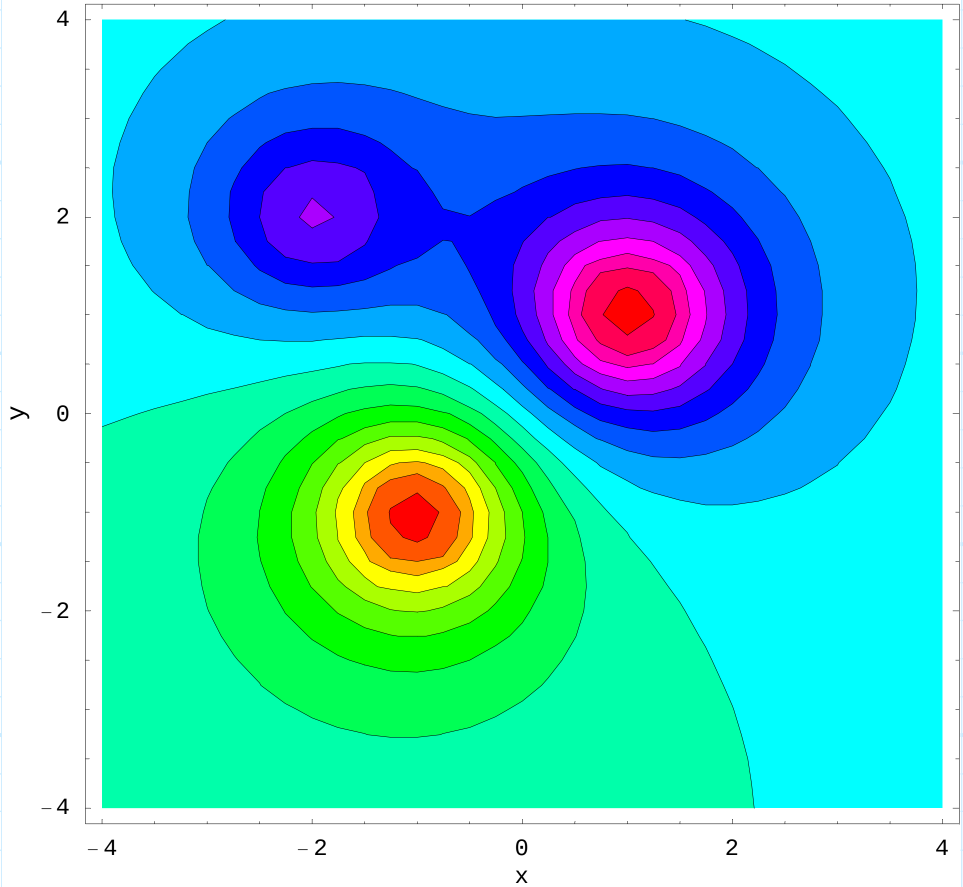


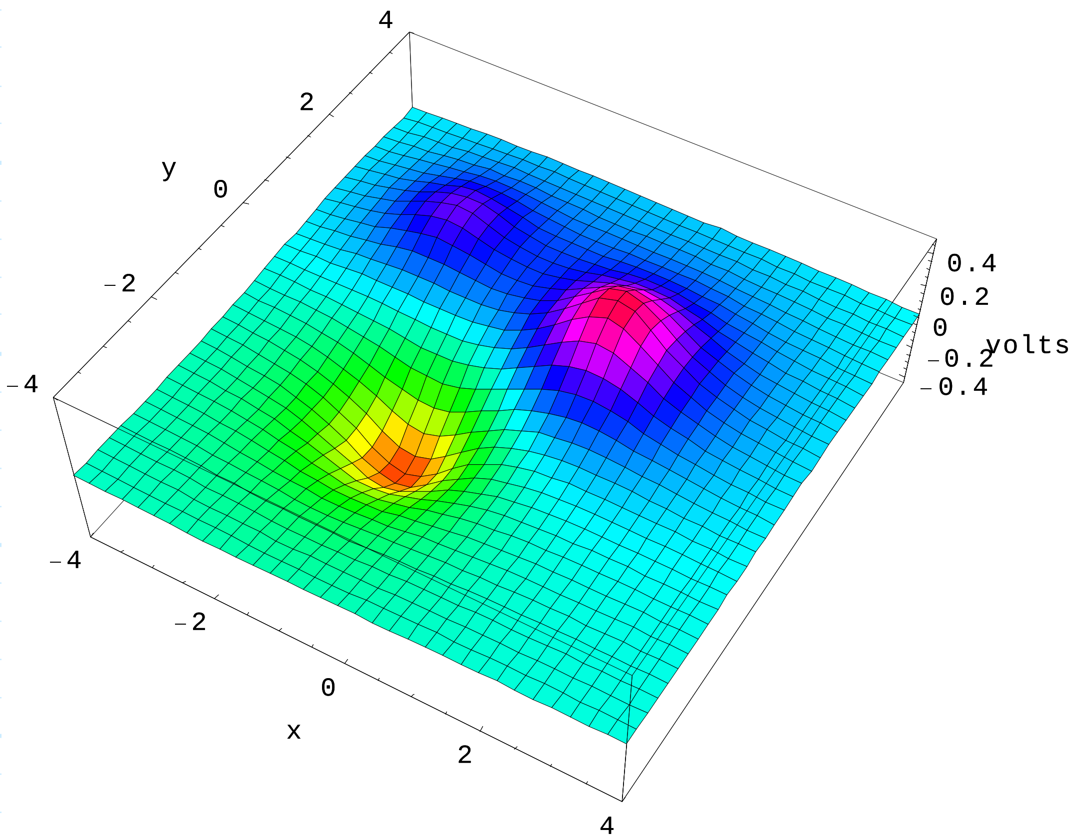
Combining the electric **field** plot with the electric **potential** plot, we get:

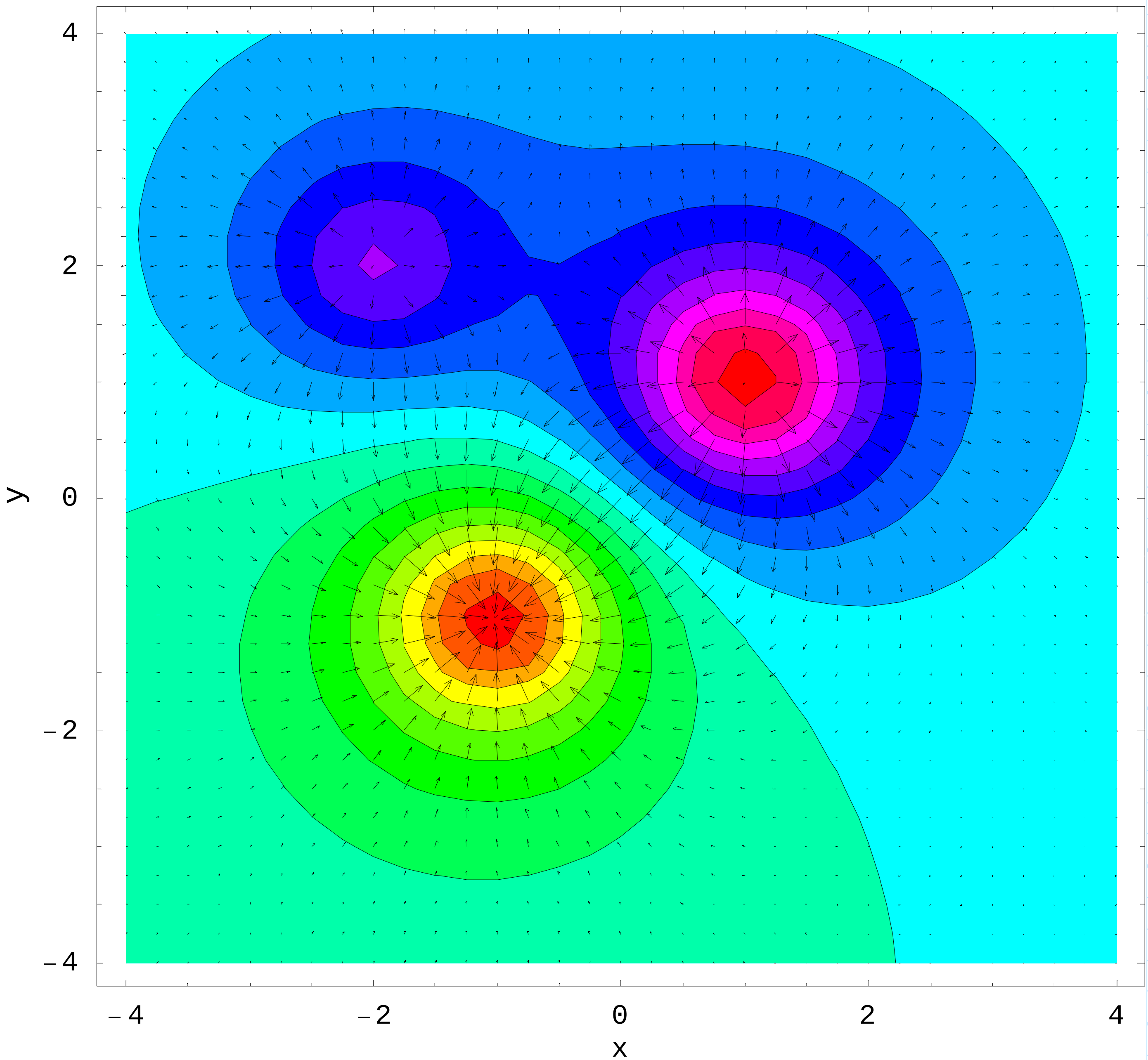


Given our understanding of the **gradient**, the above plot makes perfect sense! Do **you** see why ?

Now lets examine another **example**, where three point charges (one of them **negative!**) are present.


4
2
y 0
-2
-4
-4
-2
0
2
4
x


4
2
y
0
-2
-4
0.4
0.2
0
-0.2
-0.4
volts
-4
-2
0
x
2
4


4
2
0
−2
−4
y
−4
−2
0
2
4
x